

**Advanced Placement Calculus AB
Summer Assignment**

1. Simplify each of the following expressions:

(a) $\frac{x^3 - 4x}{x^2 - 9x + 14}$

(b) $\frac{x^2 - 4x - 32}{x^3 - 11x^2 + 24x}$

(c) $\frac{5 + x^{-1}}{25 - x^{-2}}$

(d) $\frac{\frac{2}{4} - \frac{1}{1}}{\frac{b}{b^2} - \frac{3}{9}}$

2. Rationalize the denominator in each expression:

(a) $\frac{6}{5 + \sqrt{7}}$

(b) $\frac{8}{\sqrt{6} - \sqrt{3}}$

3. In each equation, solve for x (without using a calculator):

(a) $27^{(2x)} = \left(\frac{1}{9}\right)^{x-3}$

(b) $e^{9x} = 7$

(c) $\ln x + \ln(x - 3) = \ln 4$

(d) $6^{x+5} = 36$

4. Simplify each expression:

(a) $5 \ln 6 - \ln 3$

(b) $6 \ln 2 + \ln(x^2 - 9) - \ln(x + 3)$

(c) $e^{5 \ln 3}$

5. Simplify each expression:

(a) $\log(10^{1/3})$

(b) $\ln\left(\frac{1}{e^{2r}}\right)$

(c) $4 \ln \sqrt{y} + 3 \ln \sqrt[6]{y}$

6. For each of the following equations, complete the square and reduce to one of the standard forms: $A(y - k) = (x - h)^2$ or $A(x - h) = (y - k)^2$.

(a) $f(x) = 4x^2 - 8x + 13$

(b) $x^2 - 18x + 6y + 99 = 0$

(c) $3y^2 + 42y + 16x + 19 = 0$

7. Factor each expression completely:

(a) $p^4 - 81$

(b) $125x^3 - 8$

(c) $3x^3 - 5x^2 - 75x + 125$

(d) $x^5 - 36x^3$

8. Find *all* real solutions to each equation:

(a) $125x^3 - 8 = 0$

(b) $x^5 - 36x^3 = 0$

(c) $3x^3 - 5x^2 - 75x + 125 = 0$

9. Solve for x in each equation:

(a) $\sin 2x = \sin x$; $0 \leq x \leq 2\pi$

(b) $2 \sin^2 x = 2 + \cos x$; $-2\pi \leq x \leq 2\pi$

(c) $3 \cos x = 9 \cos x \tan^2 x$; $-\infty \leq x \leq \infty$

10. Without using a calculator, evaluate the following:

(a) $\cos \frac{7\pi}{6}$

(b) $\sin \frac{3\pi}{4}$

(c) $\tan^{-1}(-\sqrt{3})$

(d) $\sin^{-1}\left(\frac{1}{2}\right)$

(e) $\sin 3\pi$

(f) $\sin^{-1}(-1)$

(g) $\tan \frac{11\pi}{6}$

(h) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

11. Solve each equation:

(a) $\frac{10 + 3x}{x} = \frac{x + 9}{x - 4}$

(b) $4x^2 + 7x + 3 = x^2$

(c) $\frac{6}{x - 3} = \frac{8x^2}{x^2 - 9} - \frac{4x}{x + 3}$

12. Find the remainder in each of the following division problems:

(a) $x^5 - 4x^4 + 3x^3 - 5x + 1$ by $x + 3$

(b) $x^5 - x^4 + 3x^3 + x^2 - 2x + 3$ by $x^3 + 1$

13. (a) The equation $6x^3 + 25x^2 + 16x - 15 = 0$ has a solution $x = -3$. Find all other solutions.

(b) Solve for x in the equation $16x^3 + 4x^2 - 4x - 1 = 0$. (All solutions are rational and between ± 1 .)

14. Solve each of the following inequalities:

(a) $x^2 + 3x - 18 \leq 0$

(b) $\frac{5x+1}{2x-3} \geq 2$

(c) $x^2 - 4x + 4 > 0$

15. Determine an equation of the following lines:

(a) The line through (-5, 2) and (2, -4).

(b) The line through (2,6) and the midpoint of the line segment from (-2, 5) to (6, -3)

(c) The line through (4, -1) and perpendicular to the line $5x + 2y + 9 = 0$.

16. Find the point of intersection of the lines: $2x - 3y + 2 = 0$ and $4x - y - 6 = 0$.

17. For the circle $x^2 - 6x + y^2 + 8y + 5 = 0$, find:

(a) The center and the radius.

(b) The equation of the tangent line at the point (1, -8).

18. A circle is tangent to the x -axis at $x = 5$ and has one y -intercept at $y = 3$.

(a) Determine the other y -intercept.

(b) Find the equation of the circle.

19. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(1, -2)$ is twice its distance from the point $B(4, -3)$. Determine the equation of the curve.

20. (a) Find the domain of the function $f(x) = \frac{5x-3}{\sqrt{x^2+2x-24}}$.

(b) Find the domain and range of the functions:

i) $f(x) = |x-3| - 5$ and ii) $g(x) = \frac{8x-1}{5x+2}$

21. Let $f(x) = \frac{|3x|}{x}$. Show that $f(x) = \begin{cases} 3, & x > 0 \\ -3 & x < 0 \end{cases}$. Find the domain and range of $f(x)$.

22. Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$, where

(a) $f(x) = 6x + 5$

(b) $f(x) = \frac{7}{x+8}$

(c) $f(x) = x^3$

23. The graph of the function $y = f(x)$ is given as follows:

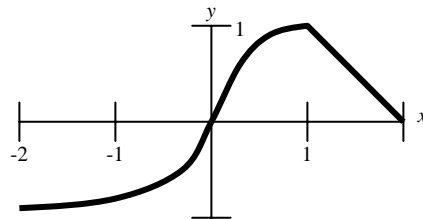
Carefully sketch a graph of each of the following

(a) $y = f(x-2)$

(b) $y = f(-x)$

(c) $y = f(|x|)$

(d) $y = |f(x)|$



24. (a) The graph of a quadratic function (a parabola) has x -intercepts -5 and 7 and a range consisting of all numbers less than or equal to 36 . Determine an expression for the function.

(b) Sketch the graph of the quadratic function $g(x) = 3x^2 - 6x - 2$.

25. Write each pair of equations as a single equation in x and y :

(a)
$$\begin{cases} x = t - 3 \\ y = t^2 + 2t \end{cases}$$

(b)
$$\begin{cases} x = \sqrt[3]{t} - 2 \\ y = t - 8 \end{cases}$$

(c)
$$\begin{cases} x = 4 \sin t \\ y = 4 \cos t \end{cases}$$

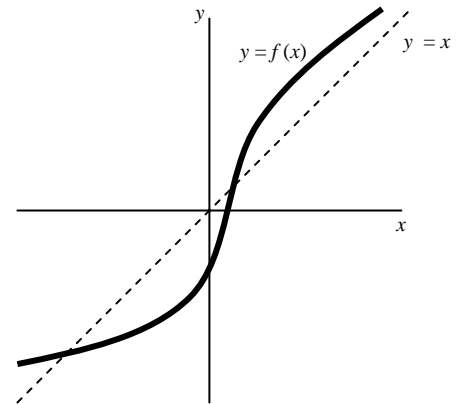
26. Find the inverse of each function:

(a) $f(x) = 7x + 4$

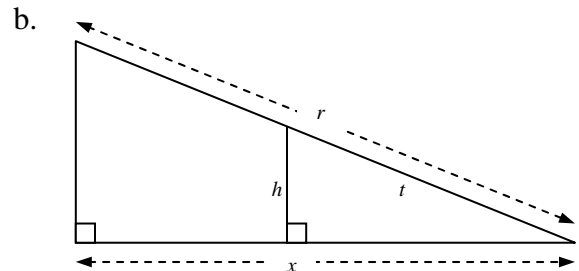
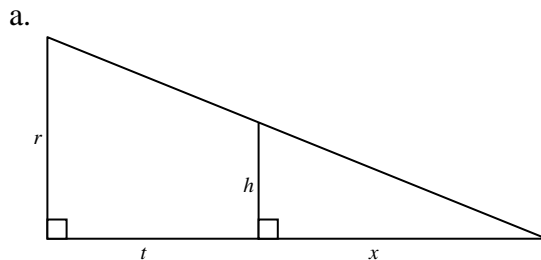
(b) $f(x) = \frac{8x + 3}{5x - 1}$

(c) $f(x) = x^2 - 10x + 27, x > 5$

27. A function $f(x)$ has the graph given below. Carefully sketch the graph of the inverse function $f^{-1}(x)$.



28. Express x in terms of the other variables in the picture.



Formula Sheet

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$
 $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ $= 1 - 2 \sin^2 x$
 $= 2 \cos^2 x - 1$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then $m = n$

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: $Ax + By + C = 0$

Name _____

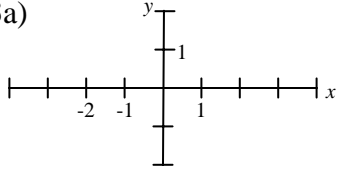
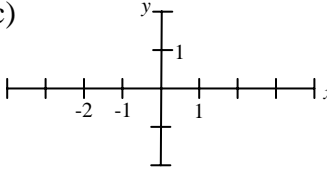
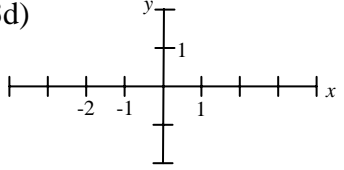
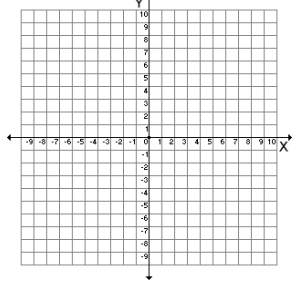
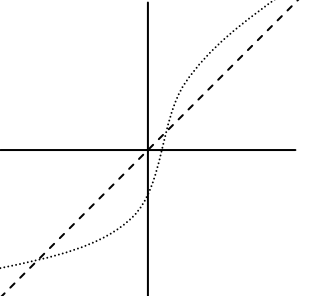
**Advanced Placement Calculus AB
Summer Packet**

Be sure to answer all questions on this answer sheet. Hand in this answer sheet with attached work and graph paper for graphing questions. The packet is due to the first day of school in September and will count as your first test grade.

1a)	1b)	1c)
1d)	2a)	2b)
3a)	3b)	3c)
3d)	4a)	4b)
4c)	5a)	5b)
5c)	6a)	6b)
6c)	7a)	7b)

7c)	7d)	8a)
8b)	8.c)	9a)
9b)	9c)	10a)
10b)	10c)	10d)
10e)	10f)	10g)
10h)	11a)	11b)
11c)	12a)	12b)

13a)	13b)	14a)
14b)	14c)	15a)
15b)	15c)	16)
17a)	17b)	18a)
18b)	19)	20a)
20b i)	20b ii)	21)
22a)	22b)	22c)

<p>23a)</p> 	<p>23b)</p> 	<p>23c)</p> 
<p>23d)</p> 	<p>24a)</p>	<p>24b)</p> 
<p>25a)</p>	<p>25b)</p>	<p>25c)</p>
<p>26a)</p>	<p>26b)</p>	<p>26c)</p>
<p>27)</p> 	<p>28 a)</p>	<p>28 b)</p>